

CORE MATHEMATICS (C) UNIT 1 TEST PAPER 4

1. Simplify as far as possible: (i) $\left(3^{\frac{1}{2}} + 3^{-\frac{1}{2}}\right)\left(3^{\frac{3}{2}} - 3^{\frac{1}{2}}\right)$, (ii) $(4^{-3})^{\frac{1}{3}}$. [4]

2. A ball is thrown from a point O . After t seconds its distance from O is x m, where $x = 40t - 5t^2$. Find the rate of change of x with time after 3 seconds. [4]

3. Differentiate with respect to x :
(i) $\frac{1}{2x} - \sqrt{x}$, (ii) $\frac{x^2 + 3}{2x^2}$. [7]

4. A rectangular garden is to have length x m, where $x > 0$. The width of the garden must be 4 m less than its length.

The perimeter of the garden cannot be more than 36 m and the area must be at least 60 m^2 .

(i) Form a linear inequality and a quadratic inequality in x . [4]

(ii) Solve your inequalities to find the range of allowable values of x . [5]

5. (i) Sketch on one diagram the straight line $y = 4 - x$ and the curve $y = \frac{1}{2x}$. [5]

The line intersects the curve at the points P and Q .

(ii) Show that the x -coordinates of P and Q are $2 + a$ and $2 - a$, where a is an irrational number to be found. [5]

6. The straight line $4y + 3x = 7$ is the tangent at the point $P(1, 1)$ to a circle with centre C .

(i) Find an equation of the straight line which passes through P and C . [4]

(ii) Given that the x -coordinate of C is 4, find the y -coordinate of C . [2]

(iii) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. [5]

7. (i) Express $x^2 - 4kx + 9$ in the form $(x + p)^2 + q$ where p and q are given in terms of k . [4]

(ii) Find the roots of the equation $x^2 - 4kx + 9 = 0$ in terms of k and deduce the set of values of k for which these roots are real and distinct. [5]

(iii) Find the exact roots of the equation $x^2 - 4kx + 9 = 0$ in the case $k = \sqrt{3}$. [3]

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8. The curve C has equation $y = x^2 - 5x + 7$.
- (i) Find the coordinates of the stationary point on C . [3]
 - (ii) Find an equation of the normal to C at the point where $x = 1$. [5]
 - (iii) Calculate the coordinates of the points where this normal intersects C again. [7]



CORE MATHS 1 (C) TEST PAPER 4 : ANSWERS AND MARK SCHEME

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|-----------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|----|
| 1. (i) $9 - 1 = 8$ | (ii) $4^{-1} = 1/4$ | M1 A1 M1 A1 | 4 |
| 2. $dx/dt = 40 - 10t$ | When $t = 3$, rate of change = 10 m/s | M1 A1 M1 A1 | 4 |
| 3. (i) | $\frac{d}{dx} \left(\frac{1}{2}x^{-1} - x^{1/2} \right) = -\frac{1}{2}x^{-2} - \frac{1}{2}x^{-1/2} = -\frac{1}{2} \left(\frac{1}{x^2} + \frac{1}{\sqrt{x}} \right)$ | B1 B1 M1 A1 | |
| (ii) | $\frac{d}{dx} \left(\frac{1}{2} + \frac{3}{2}x^{-2} \right) = -3x^{-3} = -\frac{3}{x^3}$ | M1 A1 A1 | 7 |
| 4. (i) | Width = $x - 4$ $4x - 8 \leq 36$ $x(x - 4) \geq 60$ | M1 A1 M1 A1 | |
| (ii) | $4x \leq 44$ so $x \leq 11$ $x^2 - 4x - 60 \geq 0$ | B1 M1 A1 | |
| | $(x + 6)(x - 10) \geq 0$, so $x \geq 10$ Hence $10 \leq x \leq 11$ | M1 A1 | 9 |
| 5. (i) | Line sketched, and curve with asymptotes $x = 0, y = 0$ | B2 B3 | |
| (ii) | Intersect where $2x(4 - x) = 1$ $2x^2 - 8x + 1 = 0$ | M1 A1 | |
| | $2(x - 2)^2 - 7 = 0$ $x = 2 \pm \sqrt{7/2}$, so $a = \sqrt{7/2}$ | M1 A1 A1 | 10 |
| 6. (i) | PC is perp. to tangent so has gradient $4/3$ $y - 1 = 4/3(x - 1)$ | M1 A1 M1 A1 | |
| (ii) | $x = 1 + 4/3(2) = 4$ $x = 5$ at C | M1 A1 | |

(ii)	$y - 1 = 4/3 (3) = 4$	$y = 5$ at C	M1 A1	
(iii)	Radius = distance from (1, 1) to (4, 5) = 5		M1 A1	
	$(x - 4)^2 + (y - 5)^2 = 25$	$x^2 + y^2 - 8x - 10y + 16 = 0$	M1 A1 A1	11
7.	(i) $x^2 - 4kx + 9 = (x - 2k)^2 + (9 - 4k^2)$		M1 M1 A1 A1	
	(ii) $x = 2k \pm \sqrt{4k^2 - 9}$, real and distinct for $4k^2 > 9$ $k < -3/2, k > 3/2$		M1 A1 M1 A1 A1	
	(iii) When $k = \sqrt{3}$, $x = 2\sqrt{3} \pm \sqrt{3} = \sqrt{3}$ or $3\sqrt{3}$		M1 A1 A1	12
8.	(i) $2x - 5 = 0$ when $x = 2.5$		Stat. point (2.5, 0.75)	M1 A1 A1
	(ii) At (1, 3), gradient = -3		Normal is $y - 3 = 1/3 (x - 1)$	B1 M1 A1 M1 A1
	(iii) $3y = x + 8$	$3(x^2 - 5x + 7) = x + 8$	$3x^2 - 16x + 13 = 0$	M1 A1
	$(3x - 13)(x - 1) = 0$	$x = 1, x = 13/3$		M1 A1 A1
	When $x = 13/3, y = (13/3 + 8)/3 = 37/9$		Point is (13/3, 37/9)	M1 A1 I5